

### Exercise

1. If  $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Show that  $f$  is not differentiable at  $x=0$ .

2. Show that the function  $f(x) = x|x|$  is differentiable at origin.

3. Discuss the differentiability of the following function

(I)  $f(x) = \begin{cases} 2, & x \leq 1 \\ x, & x > 1 \end{cases}$  at  $x=1$

(II)  $f(x) = \begin{cases} x \tan^{-1} \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$  at  $x=0$

(III)  $f(x) = |x-1| + |x-2|$  at  $x=1$  and  $x=2$

(IV)  $f(x) = \begin{cases} e^{-1/x^2} \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$  at  $x=0$

4. Find  $L f'(0)$  and  $R f'(0)$  for the following functions

$$(I) \quad f(x) = \begin{cases} \frac{x(e^{1/x} - 1)}{(e^{1/x} + 1)}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

$$(II) \quad f(x) = \begin{cases} \frac{x}{1 + e^{1/x}}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

$$(III) \quad f(x) = \begin{cases} \frac{x(1 - e^{-2/x})}{(1 + e^{-2/x})}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

(5) Examine the function  $f$ , where  $f(x) = \begin{cases} x^m \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$   
for derivability at  $x=0$

### ANSWERS

3. (I) not differentiable (II) not differentiable

(III) not differentiable at  $x=1$  &  $x=2$  (IV) differentiable

4. (I)  $L f'(0) = -1, R f'(0) = 1$  (II)  $L f'(0) = 1, R f'(0) = 0$

(III)  $L f'(0) = -1, R f'(0) = 1$

(5)  $f$  is differentiable for  $m > 1$